QP Set I Solution SEIT AM-IV

Q1. a

X	Y	R_1	R_2	d_i	d_i^2
40	46	4	6	-2	4
42	43	5	3.5	1.5	2.25
45	44	6	5	1	1
35	39	1	1	0	0
36	40	2	2	0	0
39	43	3	3.5	-.5	.25
					7.5

$$R = \rho = 1 - \frac{6\left[\Sigma d_i^2 + \frac{1}{12} m_1(m_1^2-1)\right]}{n(n^2-1)}$$

$$= 1 - \frac{6(7.5 + .5)}{6(36-1)} = .771$$

b. $\int_{-\infty}^{\infty} f(x)\,dx = 1$ pdf.

$k\int_0^1 (x-x^2)\,dx = 1$ $k = 6.$

Mean $= E[X] = \int_0^1 x f(x)\,dx$

$= 6\int_0^1 (x^2 - x^3)\,dx = 1/2$

$E[X^2] = \int_0^1 x^2 f(x)\,dx$

$= 6\int_0^1 (x^3 - x^4)\,dx = 3/10.$

$V(X) = E(X^2) - (E(X))^2 = \frac{3}{10} - \frac{1}{4} = 1/20.$

c. $n = 50$

$H_0 : \mu = 1800$

$H_1 : \mu > 1800.$ One-tailed test

Z_α at 1%. $= 2.33.$

$Z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1850 - 1800}{100/\sqrt{50}} = 3.54$

$\therefore$ $Z > Z_\alpha$. H_0 Rejected

ie we may support the claim of increase in breaking strength.

Q1. d.

	Opinion Favoured	Not favoured	Total
UG	290	110	400
PG	310	90	400
	600	200	800 = N.

Under H_0: Opinion is ind of the level of classes

O_i	E_i	$(O_i - E_i)^2/E_i$
290	300	·33
310	300	0.33
110	100	1.00
90	100	1.00
		2.66

Test statistic

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$= 2.66$

Tab. value of $\chi^2_{(r-1)\times(s-1)}$

$\chi^2_{1\,df}$ at 5% LOS $= 3.84$

$\therefore$ Cal $\chi^2 <$ Tab χ^2

Accept H_0

There is no association between opinion and level of classes.

e.

	C_B	Basic var B	rhs b_i	x_1	x_2	x_3	s_1	s_2	θ
			C_j	6	−2	3	0	0	
	0	s_1	2	$\boxed{2}$	−1	2	1	0	2/2 = 1 →
	0	s_2	4	1	0	4	0	1	4/1 = 4
	$Z_j = C_B x_j$		0	0	0	0	0	0	
	$\Delta_j = C_j - Z_j$			6↑	−2	3	0	0	
$R_1/2$	6	x_1	1	1	−1/2	1	1/2	0	—
$R_2 - R_1(N)$	0	s_2	3	0	$\boxed{1/2}$	3	−1/2	1	6 →
	$Z_j = C_B x_j$			6	−3	6	3	0	
	$\Delta_j = C_j - Z_j$			0	+1↑	−3	−3	0	
$R_1 + \frac{1}{2} R_2(N)$	6	x_1	4	1	0	4	0	1	
$2R_2$	−2	x_2	6	0	1	6	−1	2	
	$Z_j = C_B x_j$			6	−2	12	2	2	
	Δ_j			0	0	−12	−2	−2	

$\therefore$ all Δ_j's ≤ 0, $b_i \geq 0$. provides optimal solⁿ

$x_1 = 4$ $x_2 = 6$ Max $Z = 6(4) + 2(6) = 12$.

Q1. f.

Lagrangian funct^n

$L(x_1, x_2, \lambda) = (4x_1 + 8x_2 - x_1^2 - x_2^2) - \lambda(x_1 + x_2 - 4)$

$\frac{\partial L}{\partial x_1} = 4 - 2x_1 - \lambda$

$\frac{\partial L}{\partial x_2} = 8 - 2x_2 - \lambda$

$\frac{\partial L}{\partial \lambda} = -(x_1 + x_2 - 4)$

Solve the eq^n $\frac{\partial L}{\partial x_1} = 0$, $\frac{\partial L}{\partial x_2} = 0$ $\frac{\partial L}{\partial \lambda_2} = 0$

$4 - 2x_1 - \lambda = 0$ (1)

$8 - 2x_2 - \lambda = 0$. —(2)

$x_1 + x_2 - 4 = 0$. —(3) $\lambda = 2$ $x_1 = 1$ $x_2 = 3$.

Adding 1 & 2

$12 - 4(x_1 + x_2) - 2\lambda = 0$.

$12 - 4(4) + 2\lambda = 0$.

$X_0 = 1, 3$.

$$\Delta = \begin{vmatrix} 0 & \partial h/\partial x_1 & \partial h/x_2 \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} - \lambda \frac{\partial^2 h}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} - \lambda \frac{\partial^2 h}{\partial x_1 \partial x_2} \\ \frac{\partial h}{\partial x_2} & & \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{vmatrix} = 4.$$

Since Δ is +ive X_0 is maxima.

$Z_{max} = 18$.

Q2.a. $n = 7$ $\Sigma x = 56$ $\Sigma x^2 = 476$, $\Sigma y = 99$ $\Sigma y^2 = 1427$

$\Sigma xy = 811$

Regression lines Y on X

$y = a + bx$

$a = 8.7143$

$b = 0.6789$

$y = 8.7143 + .6786x$

Regression lines X on Y

$x = a + by$

$a = -2.0053$

$b = 0.7074$

$x = -2.0053 + .7074y$

$r = \sqrt{.6786 \times .7074} = .6928$.

Q2.d

Q2. b. $M_X(t) = E[e^{tX}]$

$= \sum e^{tx} p(x)$

$= \frac{1}{6} + \frac{1}{3}e^{t} + \frac{1}{3}e^{2t} + \frac{1}{6}e^{3t}$

$\mu_r' = \frac{\partial^r}{\partial t^r} M_X(t) \Big|_{t=0}.$

$\mu_1' = 3/2$, $19/2$ $\mu_3' = 15/2$ $\mu_4' = 115/6$

$\mu_1 = 0.$

$\mu_2' = \mu_2' - \mu_1'^2 = 11/12$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = 0$

$\mu_4 = \mu_4' - 6\mu_3'\mu_1' + 4\mu_2'\mu_1'^2 + 3\mu_1'^4 = 1.729$

Q2.c.

X	$d_1 = x_i - 17$	Y	$d_2 = y_i - 16$	d_1^2	d_2^2
19	2	15	−1	4	1
17	0	14	−2	0	4
15	−2	15	−1	4	1
21	4	19	3	16	9
16	−1	15	−1	1	1
18	1	18	2	1	4
16	−1	16	0	1	0
14	−3	–	–	9	–
$\sum x_i = 136$	0		0	36	20.

$\bar{x} = \frac{136}{8} = 17$

$\bar{y} = \frac{112}{7} = 16$

$s_1^2 = \frac{1}{n_1}\sum d_1^2 - \left(\frac{1}{n_1}\sum d_1\right)^2 = 4.5$

$s_1 = 2.1213$

$s_2^2 = \frac{1}{n_2}\sum d_2^2 - \left(\frac{1}{n_2}\sum d_2\right)^2 = 2.8571$

$s_2 = 1.6903.$

Under $H_0 : \bar{x}_1 = \bar{x}_2$

$H_1 : \bar{x}_1 \neq \bar{x}_2$ Two tailed

$|t| = \frac{\bar{x}_1 - \bar{y}_2}{\hat{s}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$= 0.93$

$\hat{s} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$

$= \sqrt{\frac{8 \times 4.5 + 7 \times 2.8571}{8+7-2}}$

$= \sqrt{4.3076} = 2.0755$

Tab t = 2.16 $\therefore$ Cal t < Tab t.

Accept H_0.

Q2.d Mean $= \bar{x} = \frac{\sum f_i x_i}{N} = \frac{\cancel{432}}{432} \quad \frac{299}{432} = 0.69$

X	f	xf_i	O_i
0	223	0	217
1	142	142	149
2	48	96	52
3	15	45	12
4	4	16	2
5	0 }19	0	0. }14
	432	299	

Under Ho fit is good

Test Statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \cancel{5.99}\ 2.6$

dof $= 6-1-1-2 = 2$ at 5% Los

$\chi^2_{Tab} = 5.99$

∴ Cal $\chi^2 <$ Tab χ^2 Accept Ho.

Fit is good.

e. Writing in standard form and adding surplus, slack, and artificial var.

Max $Z^* = -Z = -2x_1 - x_2 - M_1A_1 + 0s_1 - M_2A_2 + 0s_2$

St

$3x_1 + x_2 + A_1 = +3$

$4x_1 + 3x_2 - s_1 + A_2 = 6$

$x_1 + 2x_2 + A_2 = 3.$

$x_1, x_2, S_1, s_2 \geq 0$

			c_j	-2	-1	0	0	$-M$	$-M$	
	C_B	Basic var B	rhs b_i	x_1	x_2	s_1	s_2	A_1	A_2	θ
	$-M$	A_1	3	$\boxed{3}$	1	0	0	1	0	$3/3=1 \rightarrow$
	$-M$	A_2	6	4	3	-1	0	0	1	$6/4=3/2$
	0	s_2	3	1	2	0	1	0	$0.$	$3/1=3$
	$Z_j = C_B Y_j$			$-7M$	$-4M$	M	0	$-M$	$-M.$	
	$\Delta_j = C_j - Z_j$			$-2+7M\uparrow$	$-1+4M$	$-M$	0	0	$0.$	
$R_1/3$	-2	x_1	1	1	$1/3$	0	0		$0.$	3
$R_2-4R_{1(N)}$	$-M$	A_2	~~65~~ 2	0	$\boxed{5/3}$	-1	0		1	$6/5 \rightarrow$
$R_3-R_{1(N)}$	0	s_2	~~45~~ 2	0	$5/3$	0	1		$0.$	$6/5$
	$Z_j = C_B Y_j$			-2	$-\frac{2}{3}-\frac{5}{3}M$	M	0		$-M.$	
	$\Delta_j = C_j - Z_j$			0	$\frac{1}{3}+\frac{5}{3}M\uparrow$	$-M$	0		$0.$	
$R_1-\frac{1}{3}R_{2(N)}$	-2	x_1	$3/5$	1	0	$1/5$	0			
$3R_2/5$	-1	x_2	$6/5$	0	1	$-3/5$	0			
$R_3-\frac{5}{3}R_{2(N)}$	$0.$	s_2	$0.$	0	0	1	1			
	Z_j			-2	-1	$1/5$	$0.$			
	$\Delta_j = C_j - Z_j$			0	0	$-1/5$	$0.$			

$\because$ all Δ_j's ≤ 0. and $b_i \geq 0 \ \forall i$

Provides optimal sol^n^

$x_1 = 3/5 \quad x_2 = 6/5 \quad s_2 = 0.$

Max $Z^* = -12/5$

Min $Z = 12/5$.

Q f. Rewrite the NLPP as

$f(x_1, x_2) = x_1^2 + x_2^2$

$h_1(x_1, x_2) = x_1 + x_2 - 4$

$h_2(x_1, x_2) = 2x_1 + x_2 - 5$

Kuhn-Tucker conditions are

$\frac{\partial f}{\partial x_1} - \lambda_1 \frac{\partial h_1}{\partial x_1} - \lambda_2 \frac{\partial h_2}{\partial x_1} = 0$ $\qquad 2x_1 - \lambda_1 - 2\lambda_2 = 0 \quad (1)$

$\frac{\partial f}{\partial x_2} - \lambda_2 \frac{\partial h_1}{\partial x_2} - \lambda_1 \frac{\partial h_2}{\partial x_2} = 0$ $\qquad 2x_2 - \lambda_2 - \lambda_1 = 0 \quad (2)$

$\frac{\partial h}{\partial \lambda}$ $\quad \lambda_1 (x_1 + x_2 - 4) = 0 \quad (3)$

$\lambda_2 (2x_1 + x_2 - 5) = 0 \quad (4)$

$x_1 + x_2 - 4 \leq 0 \quad (5)$

$2x_1 + x_2 - 5 \leq 0 \quad (6)$

$x_1, x_2 \geq 0 \quad (7)$

$\lambda_1 \; \lambda_2 \geq 0 \quad (8).$

Now depending upon λ_1, λ_2 following cases arise.

1. Case $\lambda_1 = 0 \quad \lambda_2 = 0$.

From eq 1, 2.

$2x_1 = 0 \quad \boxed{x_1 = 0} \quad , \quad 2x_2 = 0 \quad \boxed{x_2 = 0}$

Provides trivial solⁿ.

2. If $\lambda_1 = 0$ & $\lambda_2 \neq 0$.

From eq 1, 2.

$2x_1 = 2\lambda_2 \qquad 2x_2 = \lambda_2$

$2x_1 + x_2 - 5 = 0$ from (4).

$\Rightarrow 2\lambda_2 + \frac{\lambda_2}{2} - 5 = 0. \qquad \lambda_2 = 5$

$\therefore x_1 = 5 \qquad x_2 = 5/2$

But condition (5) & (6) are not satisfying

Hence reject these values.

3. If $\lambda_1 \neq 0$ & $\lambda_2 = 0$

From eqⁿ (1) & (2); $2x_1 = \lambda_1$, $2x_2 = \lambda_1$

$\therefore x_1 = x_2$

From (3) $x_1 + x_2 - 4 = 0$.

Put $x_1 = x_2 \qquad 2x_1 - 4 = 0 \qquad x_1 = 2$

$x_2 = 2$.

$\lambda_1 = 4$.

Satisfying condition 5, 6, 7, 8.

$Z_{max} = 8.$

Case 4 : $\lambda_1 \neq 0, \lambda_2 \neq 0$.

From (3) and (4)

$x_1 + x_2 - 4 = 0$.

$2x_1 + x_2 - 5 = 0$.

Solving $x_1 = 1 \quad x_2 = 3$.

From 1 & 2. $\lambda_1 + 2\lambda_2 = 2$

$\lambda_1 + \lambda_2 = 6$

$\lambda_1 = 10 \quad \lambda_2 = -4$.

$\lambda_1 \geq 0$ but $\lambda_2 < 0$ Rejecting these values.

$\therefore$ Reqd. solution $x_1 = 2, x_2 = 2$ $Z_{max} = 8$.

Q3.a.

X	$X - \bar{X} = x'$	x'^2	y	$y' = y - \bar{y}$	y'^2	$x'y'$
28	−8	64	23	−8	64	64
45	9	81	34	3	9	27
40	4	16	33	2	4	8
38	2	4	34	3	9	6
35	−1	1	30	−1	1	1
33	−3	9	26	−5	25	15
40	4	16	28	−3	9	−12
32	−4	16	31	0	0	0
36	0	0	36	5	25	0
33	−3	9	35	4	16	−12
		216			162	97

$\bar{X} = \frac{360}{10} = 36$

$\bar{Y} = \frac{310}{10} = 31$

$$r = \frac{\Sigma x'y'}{\sqrt{\Sigma x'^2}\sqrt{\Sigma y'^2}} = \frac{97}{\sqrt{162 \times 216}} = 0.5186$$

Sol^n $p_1 = P($item produced by $A) = 0.3$

$p_2 = P($ $B) = 0.5$

$p_3 = P($ $C) = 0.2$

Let D be event that item is defective

$p_1' = P(D|A) = .8 \quad p_2' = P(D|B) = 0.5 \quad p_3' = P(D|C) = .1$

Bayes Th

$$P(A|D) = \frac{p_1 p_1'}{P(D)} = \frac{p_1 p_1'}{p_1 p_1' + p_2 p_2' + p_3 p_3'}$$

$$= \frac{.3 \times .8}{.3 \times .8 + .5 \times .5 + .2 \times .1} = \frac{.24}{.51} = .47$$

Q3. c. $p = \frac{0.01}{100} = .0001$ $n = 1000$

$\lambda = np = 0.1$

$P(X=x) = \frac{e^{-.1}(.1)^x}{x!}$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= e^{-.1}\left[(.1)^0 + \frac{.1}{1} + \frac{(.1)^2}{2}\right] = .9998.$

d. ~~X 578 572 570 568 572 570 570 572 596~~

X	$d = X - 580$	d^2
578	-2	4
572	-8	64
570	-10	100
568	-12	144
572	-8	64
570	-10	100
570	-10	100
572	-8	64
596	16	256
584	4	16
	-48	864

$\bar{X} = A + \frac{\sum d_i}{n}$

$= 580 - \frac{48}{10} = 575.2$

$s^2 = \frac{1}{n}\sum d_i^2 - \left(\frac{\sum d_i}{n}\right)^2$

$=$ ~~63.36~~ $68.16.$

$H_0: \mu = 577$ kg $H_1: \mu \neq 577$ kg two-tailed test

$\alpha =$ Los $= 5\%$

Test statistic $|t| = \frac{\bar{X} - \mu}{s/\sqrt{n-1}} = -0.65$

Tab t_9 dof at 5% Los $= 2.25$

$\therefore$ Cal t < Tab t

Accept Ho at α% Los.

The mean breaking strength of wire is 577 kg.

e) Rewrite LPP

Min Max $Z^* = -Z = -6x_1 - x_2$

Subject to $-2x_1 - x_2 \leq -3$

$-x_1 + x_2 \leq 0$

Adding slack variable to the constraints

Max $Z^* = -Z = -6x_1 - x_2$

st $-2x_1 - x_2 + s_1 = -3$

	C_B	Basic var B	C_j rhs b_i	-6 x_1	-1 x_2	0 s_1	0 s_2	~~Out~~ In going ve min Δ_j/a_{ij}
←	0	s_1	-3	-2	$\boxed{-1}$	1	0	min $\left\{\frac{-6}{-2}, \frac{-1}{-1}\uparrow\right\}$ $3, 1$
	0	s_2	0	-1	1	0	1	
		$Z_j = C_B x_j$		0	0	0	0	
		$\Delta_j = C_j - Z_j$		-6	$-1\uparrow$	0	0	
	-1	x_2	3	2	1	-1	0	min $\left\{\frac{-4}{-3}\right\}$
$R_2 - R_{1(n)}$ ←	0	s_2	-3	$\boxed{-3}$	0	1	1	
		$Z_j = C_B x_j$		-2	-1	1	0	
		$\Delta_j = C_j - Z_j$		$-4\uparrow$	0	-1	0	
$R_1 - 2R_{2(n)}$	-1	x_2	1	0	1	$-4/3$	$2/3$	
$-R_2/3$	-6	x_1	1	1	0	$-1/3$	$-4/3$	
		$Z_j = C_B x_j$		-6	-1	$7/3$	$4/3$	
		$\Delta_j = C_j - Z_j$		0	0	$-7/3$	$-4/3$	

$\therefore$ all Δ_j's ≤ 0 & $b_i \geq 0 \; \forall i$

Provides optimal solⁿ for $x_1 = 1$, $x_2 = 1$.

Min $Z = 6 + 1 = 7$.